2216 *KSME International Journal, Vol. 18, No. 12, pp. 2216~2224, 2004*

Dynamic Behavior of Cracked Pipe Conveying Fluid with Moving Mass Based on Timoshenko Beam Theory

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In this paper we studied about the effect of the open crack and the moving mass on the dynamic behavior of simply supported pipe conveying fluid. The equation of motion is derived by using Lagrange's equation and analyzed by numerical method. The crack section is represented by a local flexibility matrix connecting two undamaged pipe segments i.e. the crack is modeled as a rotational spring. The influences of the crack severity, the position of the crack, the moving mass and its velocity, the velocity of fluid, and the coupling of these factors on the vibration mode, the frequency, and the mid-span displacement of the simply supported pipe are depicted.

Key Words : Dynamic Behavior, Open Crack, Moving Mass, Pipe Conveying Fluid, Timoshenko Beam Theory, Flexibility Matrix, Crack Severity

Nomenclature

a_c : Depth of crack
A : Cross-sectional area
b : Half-length of crack
C : Flexibility matrix
EI : Bending stiffness
J : Strain energy density function
K_I : Stress intensity factor (fracture mode I)
K_R : Rotating spring coefficient
k : Number of segment
L : Length of pipe
m : Mass per unit length of pipe
m_f : Fluid mass per unit length of pipe
m_m : Moving mass
q : Deflection of pipe
t_p : Thickness of pipe
u : Velocity of fluid
U : Velocity of fluid, dimensionless
v : Velocity of moving mass
w : Deflection of pipe, dimensionless
θ : Half-angle of crack
θ^* : Half-angle of crack, dimensionless
κ : Shearing coefficient of cross-section
ξ : Distance measured along pipe, dimensionless

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(Manuscript **Received** June 22, 2004; **Revised** September 22, 2004)

1. Introduction

Cracks are present in structures due to various reasons. The detection and control of damage in mechanical structures are an important concerns of engineering communities. When a structure is subjected to damage its dynamic response is varied due to the change of its mechanical characteristics. Our interesting issue is the effect of an open crack on the structural response. And the effect of moving mass on the structures and the machines is an important problem both in the field of transportation and in the design of machining processes. And the fluid that flow inside the pipe acts as the concentrated tangential follower force at the tip of a pipe, and exert a lot of influences on dynamic characteristic of a pipe. Therefore, dynamic behavior of fluid-conveying pipes has formed the subject of a large number of papers since the early 1960s. The transfer of energy between the flowing fluid and the pipe was discussed by Benjamin (1961). Sugiyama and Langthjem (1999) studied the dynamic stability of a cantilevered two-pipe system conveying different fludis. Lee (1996) studied the dynamic response of a clamped-clamped beam acted upon by a moving mass. He analyzed the problem of the moving mass separating from the beam by monitoring the contact forces between them. A lot of studies about the dynamic behavior of the beam structure under the moving load and the moving mass was reported (Stanisic, 1985 ; Lee, 1995 ; Yoon et al., 2003). Recently, Mahmoud (2002) used an equivalent static load approach to determine the stress intensity factors for a single- or double-edge crack in a beam subjected to a moving load. Lim et al.(2003) conducted the nonlinear dynamic analysis of a cantilever tube conveying fluid with system identification. Chondros and Dimarogonas (1980) studied the effect of the crack depth on the dynamic behavior of a cantilevered beam. They showed that increasing the crack depth reduces the natural frequency of the beam. Ostachowicz and Krawczuk (1991) investigated the influence of the position and the depth of two open cracks upon the fundamental frequency of the natural flexural vibrations of a cantilever beam. To model the effect of the local stress in the crack, they introduced two different functions according to the symmetry of the crack. The dynamic characteristics of a cracked rotor supported on AMBs are studied ; the effect of using optimal controller parameters on the dynamics of the active cracked rotor and the effects of crack on the control system are analyzed (Zhu et al., 2003).

In this study, the crack effects on the dynamic behavior of the cracked pipe conveying fluid with the moving mass are investigated. That is, the influences of a crack, position of a crack and the velocity of the moving mass have been studied on the dynamic behavior of a simply supported pipe conveying fluid. The simply supported pipe conveying fluid has a circular hollow cross-section. The crack is assumed to be always open during the vibrations.

2. The Theory and Formulations

We consider a uniform pipe of length L applying the Timoshenko beam theory. The system with a moving mass on the cracked simply supported pipe conveying fluid is shown in Fig. 1(a), where m_m is the moving mass, v is the velocity of moving mass and u is the velocity of fluid flow. The flow velocity u is assumed constant, the fluid has a constant mass per unit length of pipe. And L is the total length of the pipe, x_c is the position of the crack. Figure 1(b) shows a circular hollow cross-section of the cracked section. θ_c and $2b$ are the crack depth (severity) and the length of a crack, respectively. Two equations of motion are derived for the two parts of the pipe located on the left and on the right of the cracked section.

2.1 The energy of a pipe and moving mass

By using the assumed mode method, the lateral displacement $y(x, t)$ of a simply supported pipe and rotation $\theta(x, t)$ in xy plane respectively, can be assumed to be as

$$y(x, t) = \sum_{n=1}^{\mu} \phi_n(x)\, q_n(t) \tag{1}$$

$$\theta(x, t) = \sum_{n=1}^{\mu} \psi_n(x)\, d_n(t) \tag{2}$$

where $q_n(t)$ and $d_n(t)$ are generalized coordinates, which are time dependent, μ is the total number of the generalized coordinates. $\phi_n(x)$ and $\psi_n(x)$ are the spatial mode functions of a simply supported pipe when there is without the fluid and a moving mass (Zhu et al., 1999).

$$\begin{aligned} \phi_n(x) &= B_n \sin\left(\frac{n\pi x}{L}\right) \\ \psi_n(x) &= \cos\left(\frac{n\pi x}{L}\right) \end{aligned} \tag{3}$$

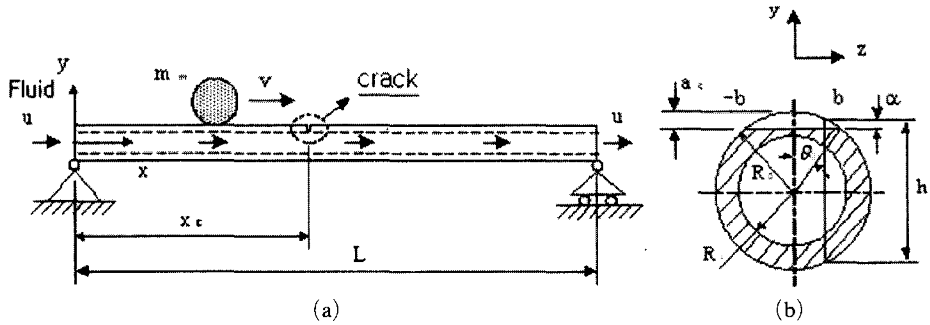


Fig. 1 Geometry of the cracked simply supported pipe with the moving mass

where

$$B_n=\frac{n\pi L}{(n\pi)^2-b_n^2 s^2},\ s^2=\frac{EI}{\kappa AGL^2},\ r^2=\frac{I}{AL^2}$$

$$b_n^2=\frac{1+(n\pi)^2(r^2+s^2)-\sqrt{(1+(n\pi)^2(r^2+s^2))^2-4(n\pi)^2r^2s^2}}{2r^2s^2}$$

where G is the shear modulus of the pipe material, A is the cross-sectional area, E is the Young's modulus, and I is the moment of the inertia of the pipe cross-section. In addition, $\kappa=\frac{2(1+\nu_p)}{4+\nu_p}$ is a shearing coefficient of circular hollow cross-section and ν_p is the Poisson's ratio.

In Figure 1, the strain energy of the cracked pipe can be written as

$$V_P=\frac{1}{2}\sum_{k=1}^{2}\int_0^{L_k}\left\{\begin{bmatrix}\frac{\partial\theta_k}{\partial x}\\ \frac{\partial y_k}{\partial x}-\theta_k\end{bmatrix}^T\begin{bmatrix}EI & 0\\ 0 & \kappa GA\end{bmatrix}\begin{bmatrix}\frac{\partial\theta_k}{\partial x}\\ \frac{\partial y_k}{\partial x}-\theta_k\end{bmatrix}\right\}dx+\frac{1}{2}K_R(\Delta y_c')^2 \quad (4)$$

where EI and κGA mean the bending stiffness and the shear stiffness respectively. According to Okamura et al. (1973), assume that a cracked member can be separated at the cracked section and that both portions can be connected by a spring with spring constant K_R, corresponding to the bending moment. In Eq. (4), the quantity

$$\Delta y_c'=\left.\frac{dy}{dx}\right|_{x_2=0}-\left.\frac{dy}{dx}\right|_{x_1=x_c} \quad (5)$$

represent the jumps in the rotation. The kinetic energy T_P of the pipe is given by

$$T_P=\frac{1}{2}\sum_{k=1}^{2}\int_0^{L_k}\left\{\begin{bmatrix}\frac{\partial y_k}{\partial t}\\ \frac{\partial\theta_k}{\partial t}\end{bmatrix}^T\begin{bmatrix}\rho A & 0\\ 0 & \rho I\end{bmatrix}\begin{bmatrix}\frac{\partial y_k}{\partial t}\\ \frac{\partial\theta_k}{\partial t}\end{bmatrix}\right\}dx \quad (6)$$

where ρ is the mass density of the material. The kinetic energy of the moving mass can be expressed as

$$T_m=\frac{1}{2}m_m\sum_{n=1}^{\mu}\sum_{k=1}^{2}\{v^2q_n^2(t)\phi_{nk}'^2(x_m)+2vq_n(t)\dot{q}_n(t)\phi_{nk}(x_m)\phi_{nk}'(x_m)+\dot{q}_n^2(t)\phi_{nk}^2(x_m)+v^2\} \quad (7)$$

where $(\cdot)$ denotes the $\partial/\partial t$, and $(')$ represents the $\partial/\partial x$ and k is number of the segments. Since the horizontal velocity of the moving mass is v, the horizontal displacement of the moving mass x_m is

$$x_m=f_m(t)=\int_0^t v\,dt \qquad (0\le x_m\le L) \quad (8)$$

2.2 The work and energy due to the fluid flow

The kinetic energy of the fluid flow inside the pipe can be expressed as

$$T_f=\frac{1}{2}m_f\sum_{n=1}^{\mu}\sum_{k=1}^{2}\left[\int_0^{L_k}\{u^2+2u\phi_{nk}(x_f)\dot{q}_n(t)+\{\phi_{nk}(x_f)\dot{q}_n(t)\}^2\}dx_f\right] \quad (9)$$
$$(x_f=ut,\ 0\le x_f\le L)$$

The work of a follower force due to the fluid discharge is divided into two kinds of work, one is the work done by conservative force component, and the other is the work done by non-conservative force component. The work W_c due to the conservative component of a tangential follower force is (Yoon et al., 2003)

$$W_c=\frac{1}{2}\sum_{n=1}^{\mu}\sum_{k=1}^{2}\int_0^{L_k} m_f u^2\{\phi'_{nk}(x_f)q_n(t)\}^2 dx_f \quad (10)$$

The work δW_{nc} due to the non-conservative component of a follower force is

$$\delta W_{nc}=-\sum_{n=1}^{\mu} m_f u^2\{\phi'_{n2}(x_f)\phi_{n2}(x_f)\}q_n(t)\,\delta q_n(t)=0 \quad (11)$$

2.3 Boundary conditions

The boundary conditions of this cracked simply supported pipe are

$$\phi_{n1}(0)=\frac{\partial^2\phi_{n1}(0)}{\partial x^2}=0,\ \phi_{n2}(L)=\frac{\partial^2\phi_{n2}(L)}{\partial x^2}=0,\ \phi_{n1}(x_c)=\phi_{n2}(x_c)$$

$$\frac{\partial^2\phi_{n1}(x_c)}{\partial x^2}=\frac{\partial^2\phi_{n2}(x_c)}{\partial x^2},\ \frac{\partial^3\phi_{n1}(x_c)}{\partial x^3}=\frac{\partial^3\phi_{n2}(x_c)}{\partial x^3} \quad (12)$$

$$\psi_{n2}(x_c)-\psi_{n1}(x_c)=\frac{EI}{K_R}\frac{\partial^2\phi_{n2}(x_c)}{\partial x^2}$$

where

$$\phi_{nk}(x)=\begin{cases}\phi_{n1}(x) & : 0\le x\le x_c\\ \phi_{n2}(x) & : x_c\le x\le L\end{cases}$$

$$\psi_{nk}(x)=\begin{cases}\psi_{n1}(x) & : 0\le x\le x_c\\ \psi_{n2}(x) & : x_c\le x\le L\end{cases}$$

2.4 Crack modeling

Consider the bending vibrations of a uniform Timoshenko beam in the plane, which is assumed to be a plane of symmetry for any cross-section. The crack is assumed to be always open. The additional strain energy due to the crack leads to flexibility coefficients expressed by the stress intensity factors. In addition, the crack produces a local additional displacement u_i between the right and left sections of the crack. According to Castigliano's theorem in the linear elastic range, these i direction displacements u_i under the action of the force P_i are given by the following expression,

$$u_i=\frac{\partial}{\partial P_i}\int_0^{a_c} J(a)\,da \quad (13)$$

where $J(a)$ is the strain energy density function. The function is

$$J(a)=\frac{1}{E^*}(K_{IM})^2 \quad (14)$$

where $E^*=E/(1-\nu_p^2)$ for the plane strain and K_I is the stress intensity factor for mode of fracture I. The local flexibility in the presence of the width $2b$ of a crack is defined by

$$C_{ij}=\frac{\partial^2}{\partial P_i\partial P_j}\left(\int_{-b}^{b}\int_0^{a_c} J(a)\,da\,dz\right) \quad (15)$$
$$\text{for } i=1,\cdots,6,\ j=1,\cdots,6$$

The stress intensity factors for bending is given by (Liu et al., 2003)

$$K_{IM}=\frac{M}{\pi R^2 t_p}\sqrt{\pi R\theta_c}\,F_b(\theta_c) \quad (16)$$

where K_{IM} denotes the opening-type mode by bending moment. $R=(R_o+R_i)/2$ is the mean radius, θ_c is the half-angle of the total crack (the crack severity will be indicated by θ_c/π as percentage) and

$$F_b(\theta_c)=1+A_t\left[4.5967\left(\frac{\theta_c}{\pi}\right)^{1.5}+2.6422\left(\frac{\theta_c}{\pi}\right)^{4.24}\right] \quad (17)$$

where

$$A_t=\left(0.125\frac{R}{t_p}-0.25\right)^{0.25} \quad \text{for } 5\le\frac{R}{t_p}\le 10$$
$$A_t=\left(0.4\frac{R}{t_p}-3.0\right)^{0.25} \quad \text{for } 10\le\frac{R}{t_p}\le 20 \quad (18)$$

where t_p is the thickness of the pipe. Substituting equations (16) ~ (18) into equation (15), the flexible matrix due to the crack can be obtained.

2.5 The equation of motion

2.5.1 The dimensionless equation of motion

By applying the Lagrange's equation to the work and energy functions, the system equation of motion is obtained. The following dimensionless parameters are introduced :

$$\xi=\frac{x}{L},\ \xi_f=\frac{x_f}{L}=uL\sqrt{\frac{m}{EI}}\tau,\ \tau=\frac{t}{L^2}\sqrt{\frac{EI}{m}},\ M_m=\frac{m_m}{mL}$$
$$U=uL\sqrt{\frac{m_f}{EI}},\ L_k^*=\frac{L_k}{L},\ \xi_c=\frac{x_c}{L},\ \beta=\frac{m_mL}{\sqrt{mEI}}\bar{v}$$
$$\gamma=\frac{m_mL^3}{EI}\bar{v}^2,\ w=\frac{q}{L},\ M_f=\frac{m_f}{m},\ \xi_m=\frac{x_m}{L}=\bar{v}L^2\sqrt{\frac{m}{EI}}\tau \quad (19)$$
$$K_R^*=\frac{K_RL}{EI},\ Q=\frac{\kappa GAL^2}{EI},\ T=\frac{mL^2}{\rho I}$$

where $\bar{v}$ is v/L. Therefore, the dimensionless equations of motion in matrix form are obtained as follows :

$$\mathbf{M}_b\ddot{\mathbf{w}}+\mathbf{C}_b\dot{\mathbf{w}}+\mathbf{K}_b\mathbf{w}=\mathbf{F}_b\mathbf{d} \tag{20}$$

$$\mathbf{M}_r\ddot{\mathbf{d}}+\mathbf{K}_r\mathbf{d}=\mathbf{F}_r\mathbf{w} \tag{21}$$

where $(\cdot)$ denotes the $\partial/\partial\tau$, the matrix of the Eqs. (20) and (21) can be written respectively as

$$\mathbf{M}_b=\sum_{n=1}^{\mu}\sum_{k=1}^{2}\left\{\int_0^{L_k^*}\phi_{nk}^2(\xi)\,d\xi + M_f\int_0^{L_k^*}\phi_{nk}^2(\xi_f)\,d\xi_f+M_m\phi_{nk}^2(\xi_m)\right\} \tag{22a}$$

$$\mathbf{C}_b=\sum_{n=1}^{\mu}\sum_{k=1}^{2}\left\{M_f\int_0^{L_k^*}\frac{d}{d\tau}\{\phi_{nk}^2(\xi_f)\}\,d\xi_f + M_m\frac{d}{d\tau}\{\phi_{nk}^2(\xi_m)\}\right\} \tag{22b}$$

$$\begin{aligned}\mathbf{K}_b=\sum_{n=1}^{\mu}\sum_{k=1}^{2}\Big[&Q\int_0^{L_k}\{\phi'_{nk}(\xi)\}^2d\xi \\ &+\beta\left\{\frac{d}{d\tau}\{\phi'_{nk}(\xi_m)\}\phi_{nk}(\xi_m)+\frac{d}{d\tau}\{\phi_{nk}(\xi_m)\}\phi'_{nk}(\xi_m)\right\}-\gamma\{\phi'_{nk}(\xi_m)\}^2 \\ &+\sqrt{M_f}U\int_0^{L_k}\left\{\frac{d}{d\tau}\{\phi'_{nk}(\xi_f)\}\phi_{nk}(\xi_f)+\frac{d}{d\tau}\{\phi_{nk}(\xi_f)\}\phi'_{nk}(\xi_f)\right\}d\xi_f \\ &-U^2\int_0^{L_k}\{\phi'_{nk}(\xi_f)\}^2d\xi_f+K_R^*\{\phi'_{n2}(\xi_2=0)-\phi'_{n1}(\xi_1=\xi_c)\}^2\Big]\end{aligned} \tag{22c}$$

$$\mathbf{F}_b=\sum_{n=1}^{\mu}\sum_{k=1}^{2}Q\int_0^{L_k^*}\phi'_{nk}(\xi)\,\psi_{nk}(\xi)\,d\xi \tag{22d}$$

$$\mathbf{M}_r=\sum_{n=1}^{\mu}\sum_{k=1}^{2}\int_0^{L_k^*}\psi_{nk}^2(\xi)\,d\xi \tag{23a}$$

$$\mathbf{K}_r=\sum_{n=1}^{\mu}\sum_{k=1}^{2}T\int_0^{L_k}\left(\left\{\frac{d\psi_{nk}(\xi)}{d\xi}\right\}^2+Q\{\psi_{nk}(\xi)\}^2\right)d\xi \tag{23b}$$

$$\mathbf{F}_r=QT\sum_{n=1}^{\mu}K\int_0^{L_k^*}\phi'_{nk}(\xi)\,\psi_{nk}(\xi)\,d\xi \tag{23c}$$

2.5.2 The modal formulation

The Eqs. (20) and (21) can be transformed into the following equation :

$$\mathbf{M}^*\dot{\boldsymbol{\eta}}+\mathbf{K}^*\boldsymbol{\eta}=0 \tag{24}$$

where

$$\mathbf{M}^*=\begin{bmatrix}\begin{bmatrix}\mathbf{M}_b & 0\\ 0 & \mathbf{M}_r\end{bmatrix} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix} \tag{25}$$

$$\boldsymbol{\eta}=[\dot{\mathbf{w}}\ \dot{\mathbf{d}}\ \mathbf{w}\ \mathbf{d}]^{\mathrm{T}}$$

where $\mathbf{I}$ represents a unit matrix. For the complex modal analysis, it is assumed that $\boldsymbol{\eta}$ is a harmonic function of τ expressed as

$$\boldsymbol{\eta}=e^{\lambda\tau}\boldsymbol{\Theta} \tag{26}$$

where λ is the eigenvalue, and $\boldsymbol{\Theta}$ is the corresponding mode shape. From the eigenvalues obtained from Eqs. (24) ~ (26), the frequencies can be obtained.

3. Numerical Results and Discussion

In this study, the dynamic behavior of the cracked simply supported pipe influenced by the moving mass, the crack severity $\theta^*(=\theta_c/\pi)$, and the position of a crack are computed by the forth order Runge-Kutta method. To illustrate this response, the length of a pipe L=0.8 m, outside-radius R_o=0.1 m and inside-radius R_i= 0.08 m were considered ($E=2.1\times10^{11}$ Pa, density=7860 kg/m^3). The numerical results were obtained for the first mode of vibration.

3.1 Results for mid-span deflection

Figure 2 shows the dimensionless mid-span deflection for a cracked pipe conveying fluid with M_m=0.3 and U=0.5. The crack position is 3/8. Generally, the mid-span deflection of a simply supported pipe is proportional to the crack severity. As the crack severity is increased, the position of the moving mass that makes the maximum mid-span deflection of the simply supported

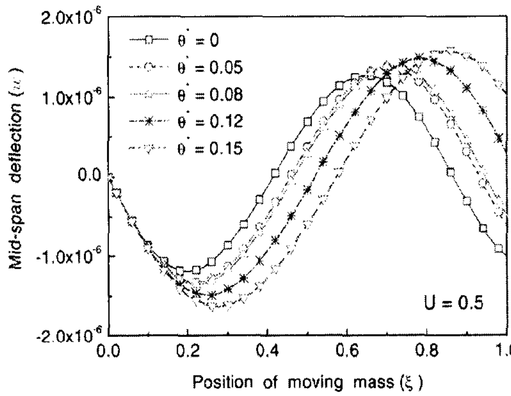
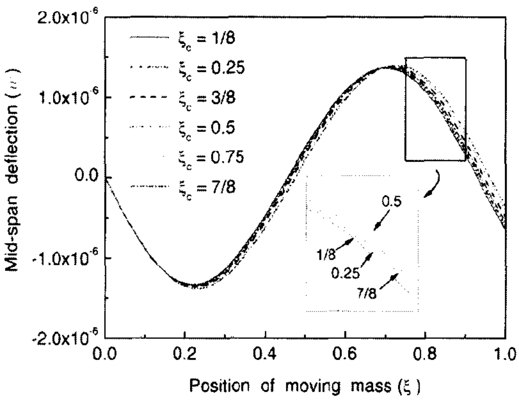
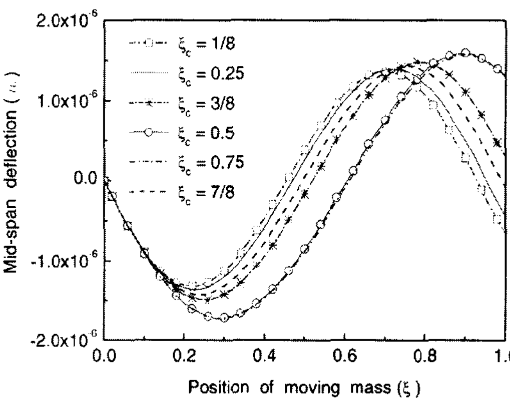


Fig. 2 Variation of mid-span deflection of a cracked pipe conveying fluid with moving mass according to crack severity (v=0.8 m/s, ξ_c= 3/8)

pipe is moved to the rear bound of the pipe. In $\theta^*=0.05$, the maximum mid-span deflection of the pipe occurs at a distance of $\xi=0.71$ from the left-hand support. In $\theta^*=0.15$, the maximum mid-span deflection of the pipe occurs at $\xi=0.85$. Figure 3 represents the variation of the mid-span deflection of a cracked pipe conveying fluid with moving mass according to the crack positions for $v=0.8$ m/s, $U=0.5$. Fig. 3(a) and (b) are the mid-span deflection of the pipe when the crack severities are 0.08 and 0.12, respectively. These results mean that when the crack position is 0.5 its effect is the largest on the mid-span deflection of the pipe. Figure 4 shows the variation of the mid-span deflection of a cracked pipe conveying fluid with moving mass according to the velocities of fluid. The mid-span deflection of a simply supported pipe is proportional to the velocity of the fluid. In Fig. 4(a), the difference of maximum mid-span deflection of the pipe in the two case of $U=0.5$ and $U=2$ is about 31.15%. In Fig. 4(b), the mid-span deflection of the pipe appears on the one-direction during the moving mass stay on the pipe. Because the velocity of moving mass is higher and the time staying on the pipe is shorter than those in Fig. 4 (b) respectively, we deduce the maximum mid-span displacement of the pipe occurs when the moving mass is leaving the pipe (Weaver et al., 1990). The variation of mid-span deflection of a cracked pipe conveying fluid

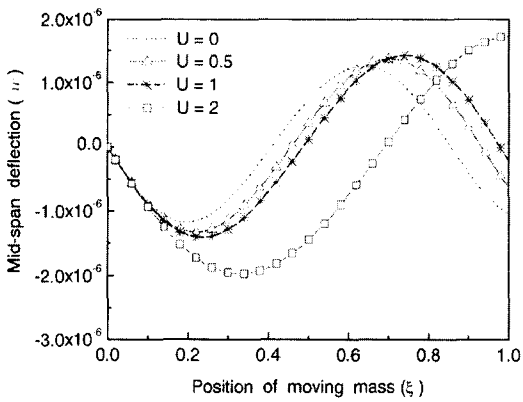


(a) Crack severity $(\theta^*)=0.08$

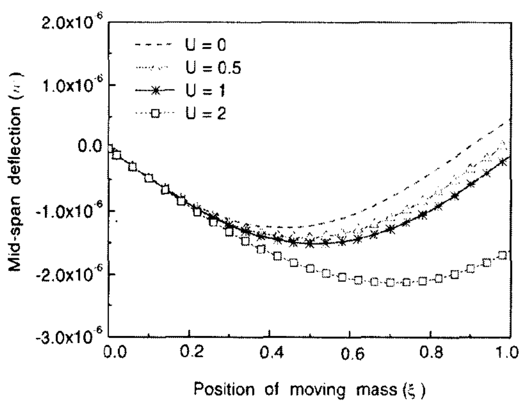


(b) Crack severity $(\theta^*)=0.12$

Fig. 3 Variation of mid-span deflection of a cracked pipe conveying fluid with moving mass according to crack position $v=0.8$ m/s)



(a) $v=0.8$ m/s



(b) $v=1.6$ m/s

Fig. 4 Variation of mid-span deflection of a cracked pipe conveying fluid with moving mass according to velocity of fluid $(\theta^*=0.08, \xi_c=3/8)$

according to the moving mass is shown in Fig. 5. In curves the crack severity, the crack position and the velocity of the moving mass are 0.08, 3/8 and 0.8 m/s, respectively. Totally, as the moving mass is increased, the mid-span deflection of simply supported pipe conveying fluid is increased. As the moving mass is increased, the position of the moving mass that appears the maximum mid-span deflection of the simply supported pipe is gradually moved to the rear bound of pipe. These are results by the coupling between the moving mass and the velocities of the moving mass. Figure 6 makes a comparison between mid-span deflection of Euler-Bernoulli beam and Timoshenko beam. When the fluid velocity is 0.5, the difference of maximum mid-span deflection of the pipe in the two case of Euler-Bernoulli beam and Timoshenko bema is about 9.77%.

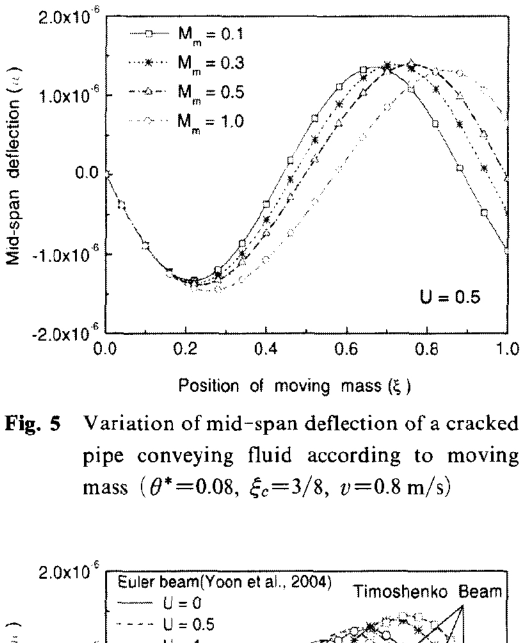


Fig. 5 Variation of mid-span deflection of a cracked pipe conveying fluid according to moving mass ($\theta^*=0.08$, $\xi_c=3/8$, $v=0.8$ m/s)

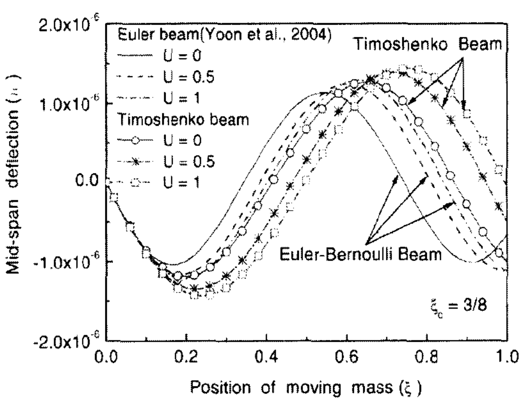
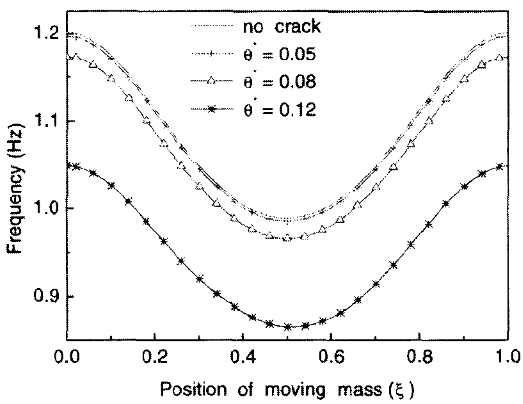


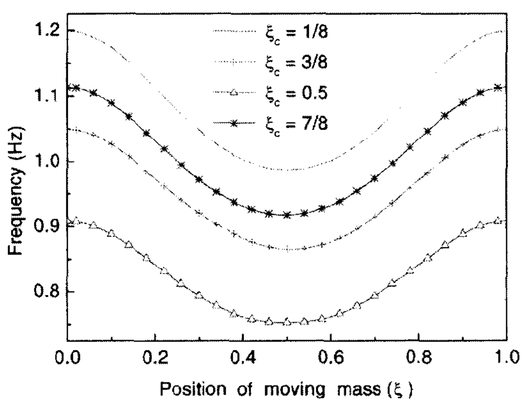
Fig. 6 Comparison between mid-span deflection of Euler-Bernoulli beam and Timoshenko beam ($\theta^*=0.05$, $\xi_c=3/8$, $v=0.8$ m/s, $M_m=0.3$)

3.2 Results for frequency

Figures 7 and 8 show the frequencies of a cracked pipe conveying fluid with the moving mass. In Fig. 7(a), the frequencies of the simply supported pipe are in inverse proportion to the crack severity. Figure 7(b) represents the frequencies of a cracked pipe conveying fluid with the moving mass according to the variation of the crack position. When the crack position exists in the center of the simply supported pipe conveying fluid, the frequency has the smallest value.



(a) variation of crack severity



(b) variation of crack position

Fig. 7 Frequencies of a cracked pipe conveying fluid with moving mass ($v=0.8$ m/s)

The difference of frequencies of the cracked pipe in the two case of $\xi_c=1/8$ and $\xi_c=7/8$ is about 6.89%. Figures 8(a) and (b) show the frequencies of a cracked pipe conveying fluid with the moving mass according to the variation of the moving mass and fluid velocity, respectively. In Fig. 8, the crack severity θ^* is 0.08, the crack position ξ_c is 3/8, and the velocity of fluid U is 0.5. In Fig. 8(a), as the moving mass is increased, the frequency of simply supported pipe conveying fluid is decreased. When the position of the moving mass exists in the center of the simply supported pipe conveying fluid, the difference of frequencies of the cracked pipe in the two case of $M_m=0$ (without moving mass) and $M_m=0.1$ is about 7.06%. And the difference of frequencies of the cracked pipe in the two case of $M_m=0.1$ and $M_m=0.3$ is about 11.37%. In Fig. 8(b), the frequencies of the simply supported pipe are in inverse proportion to the velocity of fluid.

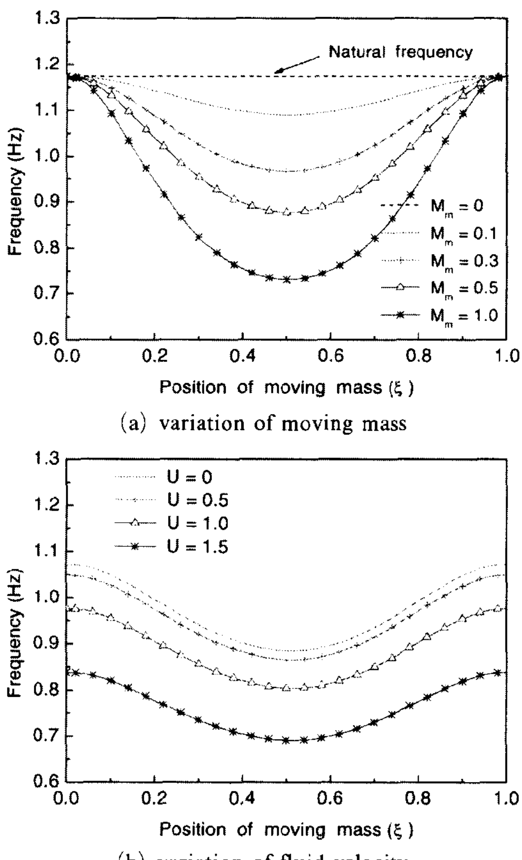


Fig. 8 Frequencies of a cracked pipe conveying fluid with moving mass ($\theta^*=0.08$, $\xi_c=3/8$)

4. Conclusions

In this paper, the influences of the crack severity and moving mass have been studied on the dynamic behavior of the cracked simply supported pipe conveying fluid by the numerical method. The cracked pipe has been treated as two undamaged segments connected by a rotational elastic spring at the crack section. The stiffness of the spring depends on the crack severity and the geometry of the cracked section. The main conclusions are the following.

(1) When the moving mass and the velocity of fluid are constant, the mid-span deflection of the cracked simply supported pipe is proportional to the crack severity.

(2) When the crack position is 0.5, its effect is the largest on the mid-span deflection of the cracked simply supported pipe conveying fluid.

(3) As the moving mass and the fluid velocity are increased, the mid-span deflection of the cracked simply supported pipe conveying fluid is increased.

(4) When the crack position exists in center of the pipe conveying fluid, its frequency has the smallest value. And totally, the frequencies of the simply supported pipe are in inverse proportion to the crack severity.

(5) The frequencies of cracked simply supported pipe conveying fluid are in inverse proportion to the fluid velocity and the moving mass, respectively.

These study results will contribute to the safety test and stability estimation of structures of a cracked pipe conveying fluid with the moving mass.

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